Exam 3 Review: Sections 3.5-3.7 and 7.1-7.5

Section 3.5 Find the general solution to the differential equation

$$y'' + 2y' + 2y = 3x^2 - 1.$$

Section 3.5 Find the general form of the particular solution for the differential equation

$$y^{(4)} - 5y'' + 4y = e^x - xe^{2x}.$$

(Do not solve for the undetermined coefficients.)

Section 3.6 Consider an undamped mass-and-spring system with mass $m = 1$, spring constant $k = 100$ and forced oscillations given by the function $F(t) = 225 \cos 5t + 300 \sin 5t$. Assume further that $x(0) = 375$ and $x'(0) = 0$. Find the solution function $x(t)$.

Sections 3.7 Consider an RLC circuit with $R = 40$ ohms, $L = 10$ henries, $C = 0.02$ farads and $E(t) = 50 \sin 2t$ volts at time t. This information gives the differential equation

$$10I'' + 40I' + 50I = 100 \cos 2t$$

for the current $I(t)$ (in amperes). Find the general complementary solution and the particular solution for this circuit.

Section 7.1 State the Laplace transform of the following functions.

1. $f(t) = 1$
2. $g(t) = e^{-3t}$
3. $h(t) = t^{11}$
4. $i(t) = \sin 4t$
5. $j(t) = \cos 5t$

Section 7.1 Find the inverse Laplace transform for the function $F(s) = \frac{3s+1}{s^2+4}$.

Sections 7.3 Use Laplace transforms to solve the initial value problem

$$x'' + 8x' + 25x = 0; \quad x(0) = 2, x'(0) = 3.$$

Section 7.2 Find the inverse Laplace transform for the function $F(s) = \frac{1}{s^2(s^2+1)}$.

Section 7.2 Find the inverse Laplace transform for the function $F(s) = \frac{5s-4}{s^3-s^2-2s}$.

Section 7.3 Find the Laplace transform for the function $f(t) = t^4 e^{\pi t}$.

Section 7.4 Use the fact that $\mathcal{L}\{(f * g)(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$ to find

$$\mathcal{L}^{-1}\{\frac{98}{(s-2)(s-3)}\}.$$

Section 7.4 Use Laplace transforms to find a non-trivial solution to the differential equation

$$tx'' + (3t-1)x' + 3x = 0; \quad x(0) = 0.$$

Section 7.5 Use the fact that $\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$ to find

$$\mathcal{L}\{f(t)\} \quad \text{where } f(t) = \begin{cases} \cos \pi t, & \text{if } 0 \le t \le 2 \\ 0, & \text{if } t > 2. \end{cases}$$

Section 7.5 Consider the differential equation

$$y^{(4)} + 2y'' + y = 4t^3 e^t; \quad y(0) = y'(0) = y''(0) = y^{(3)}(0) = 0.$$

(a) Solve for the transform $Y(s) = \mathcal{L}\{y(t)\}$.
(Hint: You may need the formula $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$ or any other method.)

(b) Find the general form of the partial fraction decomposition of $Y(s)$. *You do not need to solve for the coefficients.*